# Determination of Nonsymmetric Aerodynamics of Re-entry Missiles

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**Linear and nonlinear theories which abandon the usual relations between pitch and yaw stability derivatives are developed to account for nonsymmetric aerodynamics. Motions of a 10° cone and a 60° cone are fit using two separate procedures: the numerical integration method using parametric differentiation and a newly developed approach to determine modal amplitudes, frequencies, and damping rates by the numerical inversion of Laplace transformations to begin a differential corrections procedure. It is shown that accurate fits of nonsymmetric vehicle motions are possible and that spinning bodies of revolution, at times, must be considered nonsymmetric vehicles.**

## Nomenclature

$d$ = reference length, ft
$I_{xx}, I$ = axial and transverse moments of inertia, slug/ft$^2$
$M_\alpha$ = in-plane restoring moment stability derivative, rad$^{-1}$ $M_\alpha = C_{m_\alpha} QSd$
$N_\beta$ = out-plane restoring moment stability derivative, rad$^{-1}$ $N_\beta = C_{n\beta} QSd$
$M_q$ = in-plane damping moment stability derivative, rad$^{-1}$ $M_q = C_{mq}(d/2V)QSd$
$N_r$ = out-plane damping moment stability derivative, rad$^{-1}$ $N_r = C_{nr}(d/2V)QSd$
$M_{\dot\alpha}$ = in-plane lag moment stability derivative, rad$^{-1}$ $M_{\dot\alpha} = C_{m\dot\alpha}(d/2V)QSd$
$N_{\dot\beta}$ = out-plane lag moment stability derivative, rad$^{-1}$ $N_{\dot\beta} = C_{n\dot\beta}(d/2V)QSd$
$M_{p\beta}$ = in-plane Magnus moment stability derivative, rad$^{-2}$ $M_{p\beta} = C_{mp\beta}(d/2V)QSd$
$N_{p\alpha}$ = out-plane Magnus moment stability derivative, rad$^{-2}$ $N_{p\alpha} = C_{np_\alpha}(d/2V)QSd$
$p$ = roll rate, rad/sec
$S$ = reference area, ft$^2$
$Q$ = dynamic pressure, $\frac{1}{2}\rho V^2$, lb/ft$^2$
$\psi\theta\phi$ = modified Euler angles
$\omega 1,2$ = nutation, precession frequency, rad/sec
$\omega 1,2$ = nutation, precession frequency, rad/sec
$\lambda 1,2$ = nutation, precession damping rate, sec$^{-1}$
$K_i$ = modal amplitudes, deg
$\delta_i$ = phase angles, rad

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## Introduction

WHILE a re-entry vehicle may initially be a symmetric body of revolution, prevailing flight conditions may greatly alter this symmetry. The effect of ablation and nose bluntness may also cause differences between in-plane and out-plane stability derivatives.[1] In addition, the asymmetrical nature of the boundary layer also effects the dynamic behavior of a body of revolution rotating about its longitudinal axis. Thus, while the cone shown in Fig. 1 is physically symmetric, the boundary-layer displacement surface distorted by spin causes the outer flow to "see" a nonsymmetric vehicle.

Various researchers have attempted to describe a flow model for a spinning body of revolution at angles of attack. However, this is an extremely difficult generalization to make as the character of the boundary layer is dependent on fineness ratio, Reynolds number, yaw angle, and spin rate. Recently a new analytic model of Magnus behavior was introduced which attempts to unify and extend present knowledge.[2] This reference also contains a good review of previous efforts in the field.

The importance of these flow considerations may effect the symmetry assumptions that the flight dynamicist has been employing. A linear theory that deals with a "slightly asymmetric" vehicle was introduced in Ref. 3. This theory has been extended to the nonlinear domain by several authors.[4-6] The basis of the constitutive equations used for force and moment reactions relies heavily on the work presented by Maple and Synge.[7]

Recently, various investigators have begun to question the validity of the symmetry assumption. Levy and Tobak[8] point out that it is possible to interpret a nonsymmetric damping moment as a symmetric Magnus-type moment. Sylvester and Braun[9] report that rifling on projectiles causes unpredictable Magnus forces. Stone, et al.,[10] believe that the required form of the damping moment is in fact nonsymmetric. Jaffe,[11] after being unable to obtain reasonable fits of flight data, suggests that the force model might be deficient. The many questions raised by these investigators led to the present work.

To better understand this lack of symmetry in the dynamic flowfield, a linear theory which abandons the usual relations between pitch and yaw stability derivatives may be developed.

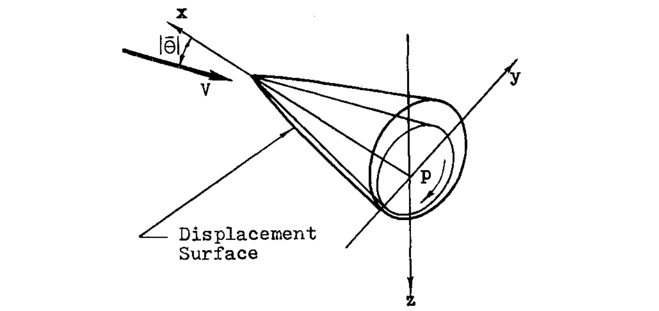


**Fig. 1 Possible effect of boundary-layer displacement surface on a spinning cone.**

It is shown that this simple linear force model can describe motions previously considered nonlinear. In addition, nonlinearities of the nonsymmetric stability derivatives with angle-of-attack dependence may be introduced if the need should arise. [12]

## Aerodynamic Theory

From the usual aymmetry considerations, a consequence of vehicles exhibiting trigonal or greater rotational symmetry arises from the fact that the aerodynamic coefficients are invariant with respect to rotation of the coordinate axes about the axis of symmetry. This has led to the use of a nonrolling aeroballistic axis system for such vehicles. Thus, a linear model, in an aeroballistic system, Fig. 2, of angular motion for vehicle with constant roll rate may be given as, [12,13]

$$\ddot{\theta}+C_1p\dot{\psi}-C_2\dot{\theta}-C_4\theta+C_6p\psi=f(t) \qquad (1a)$$

$$\ddot{\psi}-C_1p\dot{\theta}-C_3\dot{\psi}+C_5\psi-C_7p\theta=g(t) \qquad (1b)$$

where

$$\begin{array}{lll} C_1=I_{xx}/I & C_2=(M_q+M_{\dot{\alpha}})/I & C_3=(N_r-N_{\dot{\beta}})/I \\ C_4=M_\alpha/I & C_5=N_\alpha/I & C_6=M_{p\beta}/I \\ C_7=N_{p\alpha}/I & C_8=M_c/I & C_9=M_d/I \\ C_{10}=M_a/I & C_{11}=M_b=I & \phi=\int pdt \end{array}$$

$$f(t)=-C_8+C_{10}\cos\phi-C_{11}\sin\phi$$

$$g(t)=-C_9+C_{11}\cos\phi+C_{10}\sin\phi$$

with initial conditions given by

$$\begin{array}{ll} \theta(0)=C_{12} & \psi(0)=C_{14} \\ \dot{\theta}(0)=C_{13} & \dot{\psi}(0)=C_{15} \end{array}$$

Furthermore, a reduced model in which $C_{11}=C_{10}=0$ is also considered. This has been done since it has been found that it is difficult to extract the magnitude of the rolling trim moments when it is of the same order of magnitude as the noise in the data.

Equations (1) are the governing equations for the linear analysis of angular motion. However, there is one important step that has been omitted. Previously, due to rotational symmetry, certain equalities between the components of the aerodynamic derivatives were used to further simplify the equations. [7] These equalities resulted in the transformation of the two second-order equation for a complex function

$$\bar{\theta}(t)=\theta(t)+i\psi(t) \qquad (2)$$

This reduction of the linear fourth-order system to the epicyclic subset enabled closed-form solutions as functions of time and equation parameters to be obtained. [3]

Nonlinearities may be introduced into the system if it is recalled that the moments are dependent, not on the individual velocity components but rather, on the cross-velocity, $v_c$ where $v_c=(v^2+w^2)^{1/2}$ Thus, define

$$|\bar{\theta}|=(\theta^2+\psi^2)^{L} \qquad (3)$$

and nonlinear moments will be assumed to take the form of a Taylor series as suggested by Maple and Synge. [7] However, the simplifications that arise when the Maple-Synge theory is accommodated cannot readily be made since the assumption of rotational symmetry is not considered valid at this point in the development

## Numerical Methods

The basic approach utilized in this study was to obtain the aerodynamic coefficients from response data by least squares techniques. This deterministic approach was chosen as it presents a straightforward analysis of the models under consideration and the results could readily be compared with those obtained in similar analyses using a symmetric force model.

The least squares process, the matching of a mathematical model to experimental observations, is essentially the same as regression analysis. The simplest analysis can be accomplished when the model is a linear function of the parameters. However, when this is not the case, an iteration scheme must be used. The starting point for this type of fitting procedure, known as differential corrections, is basically a linearization in which a function that is linear in terms of errors in the parameters is created by means of a truncated Taylor series expansion about some approximate value of the parameters. Thus, an approximate fit to the observations is made and repeatedly corrected until some convergence criterion is met. Mathematically this requires that the system of differential equations be considered in the form

$$\dot{\boldsymbol{\Theta}}=\mathbf{F}(t,\boldsymbol{\Theta},C) \qquad (4)$$

where $\boldsymbol{\Theta}=(\theta_1,\theta_2,\ldots,\theta_m)$, $F=(F_1,\ F_2,\ldots F_m)$, and $C=(C_1,C_2,\ldots C_k)$ are vectors belonging to spaces of dimensions $m,m,$ and $k$, respectively. In addition, the $j$th element of $C$ is considered to be composed of an approximation term, $C_{jo}$, and a correction term, $\Delta C_j$.

Using the least squares criterion, a linear system of equations in $\Delta C_j$ is formed. Numerically, the values of the correction $\Delta C_j$ may be determined from this system once the values of $\partial\theta_i/\partial C_j$ can be obtained at each data point. The manner in which these partial derivatives are determined characterizes the methods to be discussed in this study.

Consider first the standard differential corrections techniques presented by Eikenberry. [14] The term "WOBBLE" has become the generic name for such an approach. In WOBBLE, analytic expressions for the partials are input to the fitting programs. To accomplish this, Eq. (4) must admit a solution of the form

$$\boldsymbol{\Theta}=\boldsymbol{\Theta}(t,C) \qquad (5)$$

In the WOBBLE program, parametric differentiation of the analytic solution is employed to obtain the partials. Thus to successfully obtain nonlinear coefficients, an asymptotic solution to the nonlinear governing equation must be formulated.

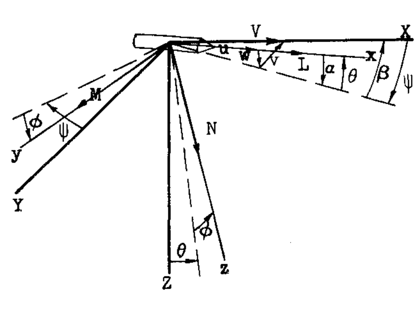


**Fig. 2 Axis system.**

Another method to evaluate the partials is used in the numeric integration method, NIM. The basic techniques employed in WOBBLE are required, only the order of application is altered. Thus, if Eq. (4) is differentiated with respect to $C_j$ parameters and initial conditions, a set of variational equations is obtained. These ordinary differential equations in time, $t$, may be numerically integrated to evaluate the partials. In this investigation, a fourth-order Runge-Kutta scheme is used. The theory employed in NIM may be found in many sources.[15-19]

While the WOBBLE program is limited to symmetric aerodynamic models, it would be advantageous to be able to extract damping rates and frequencies from the nonsymmetric models for purposes of comparison. The form of the solution of the reduced nonsymmetric model will be given by

$$\theta(t)=K_1e^{\lambda_1 t}\cos(\omega_1 t+\delta_1)+K_2e^{\lambda_2 t}\cos(\omega_2 t+\delta_2)+K_{4A} \tag{6a}$$

$$\psi(t)=K_3e^{\lambda_1 t}\cos(\omega_1 t+\delta_3)+K_4e^{\lambda_2 t}\cos(\omega_2 t+\delta_4)+K_{4B} \tag{6b}$$

However, in order to obtain approximations for fitting this model in a direct manner, it must be possible to express the amplitudes, frequencies, and damping rates as function of $\boldsymbol{C}$. Algebraically, this is not simple. Thus, consider the reduced form of the equations

$$\ddot{\theta}+C_1p\dot{\psi}-C_2\dot{\theta}-C_4\theta+C_6p\psi=-C_8 \tag{7a}$$

$$\ddot{\psi}-C_1p\dot{\theta}-C_3\dot{\psi}+C_5\psi-C_7p\theta=-C_9 \tag{7b}$$

The Laplace transform of Eqs. (7) yields expressions for $\theta^*$ and $\psi^*$ as functions of the complex frequency, $s$,

$$\theta^*(s)=N_1(s)/[sD_4(s)] \tag{8a}$$

$$\psi^*(s)=N_2(s)/[sD_4(s)] \tag{8b}$$

where $N_1(s)$ and $N_2(s)$ are fourth-order polynomials in $s$ and $D_4(s)$ is also a fourth-order polynomial, the roots of which determine the frequencies of the system.

One manner to arrive at approximations for the parameters in Eqs. (6) is based on the algebraic expressions for $\theta^*$ and $\psi^*$. If the approximate values of $C$ are inserted into $N_1(s)$, $N_2(s)$, and $D_4(s)$, the resulting polynomial fractions may be inverted numerically to determine approximations for the parameters of Eqs. (6).[20] In this study, the acronym DIFLA is used to represent this differential corrections with Laplace transforms method.

To better understand this approach, consider that an approximation for the parameter vector is given. To determine approximations for the parameters that appear in the analytic solution of the differential equation, consider the expression $X(s)=H(s)/B(s)$ where $X(s)$ is a transformed variable and $H(s)$ and $B(s)$ are polynomials. For a simple pole at $s_j$, the resulting residue $K_j$ is given by $K_j=H(s_j)/B'(s_j)$ where prime denotes differentiation with respect to complex frequency, $s$. For oscillatory motion, $s_j$ will be complex and thus $K_j$ will also be complex. Let $K_j=a_j+ib_j$, and $\bar{K}_j=a_j-ib_j$, where $a_j$ is the real part of the residue and $b_j$ is the imaginary part. Let $s_j$ also have a similar form, $s_j=\lambda_j+i\omega_j$ and $\bar{s}_j=\lambda_j-i\omega_j$.
Then for pure oscillatory motion

$$x(t)=2\sum_{j=1}^{h}e^{\lambda_j t}[a_j\cos\omega_j t-b_j\sin\omega_j t]$$

where $h$ is the number of pairs of complex conjugate poles located at $s_j$ and $\bar{s}_j$. Thus, it is seen that Eqs. (8) may be inverted to yield expressions of the form of Eqs. (6) if oscillatory motion is assumed.

In order to arrive at suitable approximations for $\boldsymbol{C}$ to be used in all of the fitting procedures being discussed, a simple least squares approach is employed. This was first suggested by Donegan and Pearson.[21] A suitable section of data is chosen over which the parameters can be evaluated once the governing equation has been written in integral form. The integrals are evaluated numerically and thus the equations are linear in the components of the parameter vector.

## Application to Aeroballistic Equations

The previously discussed fitting techniques may be applied to the aeroballistic equations, Eqs. (1) and Eqs. (7). Since the coefficients for nonsymmetric bodies are not roll invariant and therefore a body axis is usually employed, this approach will in fact test the validity of the aeroballistic approach to a so-called symmetric body of revolution. That is, since Eqs. (1) and (7) are of exactly the same form as the symmetric body equations with the exception of not having assumed certain equalities among stability derivatives, one would expect these equalities to appear in the results if they are present.

There will be three separate fitting routines employed for the linear models in this study. In summary, 1) WOBBLE uses the solution to the symmetric aeroballistic equations as a model. 2) NIM uses the nonsymmetric equations themselves as models to directly determine stability derivative coefficients. 3) DIFLA uses the solution to the nonsymmetric equations as a model to determine the oscillatory parameters. It should be noted that Eqs. (6) are similar in form to the model used in the WOBBLE routine. However, symmetry adds the constraints that $K_1=K_3$, $K_2=K_4$, $\delta_1=\delta_3+\pi/2$, and $\delta_2=\delta_4+\pi/2$. The reason for the 90° phase shift is that the equation for $\psi(t)$ in the WOBBLE model contains sine functions.

To determine the effects of symmetry on the linear frequencies, consider a body on which only nonsymmetric restoring moments are acting. Thus, Eqs. (1) become

$$\ddot{\theta}+K\dot{\psi}-m_\alpha\theta=0 \tag{9a}$$

$$\ddot{\psi}-K\dot{\theta}+n_\beta\psi=0 \tag{9b}$$

where $m_\alpha=M_\alpha/I$ $n_\beta=N_\beta/I$ $K=pI_{xx}/I$. Both classical and transform methods yield a frequency determinant which is of a biquadratic form and thus an expression for $s^2$ may be found

$$s^2=-\tfrac{1}{2}(n_\beta-m_\alpha+K^2)\pm\tfrac{1}{2}[(n_\beta+m_\alpha)^2-2K^2(m_\alpha-n_\beta)+K^4]^{\frac{1}{2}}$$

$$A\pm\tfrac{1}{2}(B)^{\frac{1}{2}} \tag{10}$$

For the symmetric case, $n_\beta=-m_\alpha$,

$$s^2=[iK/2\pm\tfrac{1}{2}(4m_\alpha-K^2)^{\frac{1}{2}}]^2$$

which gives the same frequency as the epicyclic set solution.[3]

Consider Eq. (10) to determine the values of frequency. If $B>0$ and $A<0$ and $A>\frac{1}{2}(B)^{\frac{1}{2}}$, then $s_{1,2}=i\omega_{1,2}$ and the motion will be purely oscillatory. However, if $B<0$ then

$$s^2=|A\pm i/2|B|^{\frac{1}{2}}|e^{i\theta'} \qquad \theta'=\tan^{-1}(B^{\frac{1}{2}}/2A)$$

$$s_{1,2}=|A\pm(i/2)|B|^{\frac{1}{2}}|e^{\pm i2\theta'}=\lambda_{1,2}+i\omega_{1,2}$$

so that the motion will have some damping factor along with an oscillation frequency.

This can readily be programed on a digital computer to determine how the frequencies vary due to changes in in-plane and out-plane restoring moments. Figures 3 and 4 show the variation of nutation frequency, defined as the faster of the two rates, and precession frequency as functions of a fractional percentage of $m_\alpha$. Thus, an abcissa value of 60 corresponds to an $n_\beta$ which is 60% of the chosen $m_\alpha$. The value of $K$ is taken to be unity and the behavior of the frequencies may be observed. Figure 3 shows that while the fundamental frequency drops off quite rapidly for decreasing symmetry, the change in nutation frequency is hardly noticeable. Figure 4 is for positive values of $m_\alpha$. For the given conditions symmetric theory predicts that $0<m_\alpha<0.25$ for spin stabilization. Thus, one case shown in Fig. 4, $m_\alpha=0.1$, is applicable to stable flight vehicles. Again, it is seen that the variation of nutation frequency is less than that of precession frequency over the range of abcissa values. The other case presented is for a vehicle that is not spin-stabilized. In this case both frequencies are equal.

To better discuss the effects of nonsymmetric aerodynamics, consider three sets of data which have been generated by numerically integrating Eqs. (1) using the parameters given in Table 1. Case I is purely symmetric while the other two cases represent different degrees of nonsymmetry. The accuracy of the numerical integration method approach used on each set of this generated data has been presented previously.[13]

The results of the WOBBLE fits are presented in Fig. 5. Here it is seen that while the program was able to handle the symmetric case quite well, the sectional results presenting the coefficients as functions of time yield nonlinearities in the nonsymmetric case.

A better insight is obtained by comparing fits from the same section of data when fit by both the WOBBLE routine

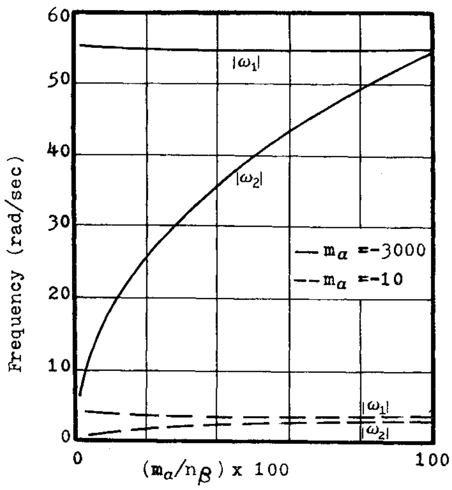


**Fig. 3 Frequency variation, statically stable case.**

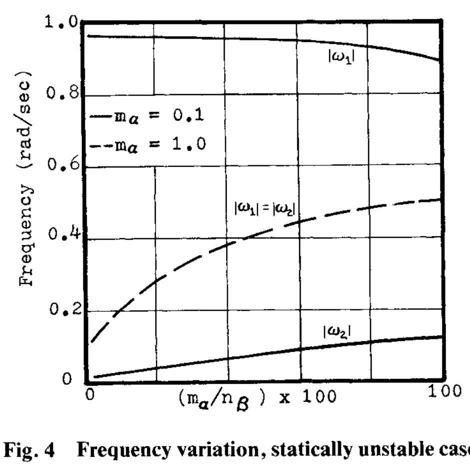


**Fig. 4 Frequency variation, statically unstable case.**

**Table 1 Parameters for generated data**

| | Case I | Case II | Case III |
|---|---|---|---|
| $C_{m\alpha}$ | $-80.0$ | $-80.0$ | $-80.0$ |
| $C_{n\beta}$ | 80.0 | 75.0 | 70.0 |
| $C_{mq}+C_{m\dot\alpha}$ | $-3000.0$ | $-3000.0$ | $-3000.0$ |
| $C_{nr}-C_{n\dot\beta}$ | $-3000.0$ | $-2500.0$ | $-2000.0$ |
| $C_{mp\beta}$ | 1000.0 | 1000.0 | 1000.0 |
| $C_{np\alpha}$ | 1000.0 | 9000.0 | 8000.0 |

$p$ = 20 rad/sec
$I_{xx}$ = 0.325 slug-ft$^2$
$I$ = 6.500 slug-ft$^2$
$V$ = 300.0 fps
$\rho$ = 0.000025 slug/ft$^3$
$d$ = 0.50 ft

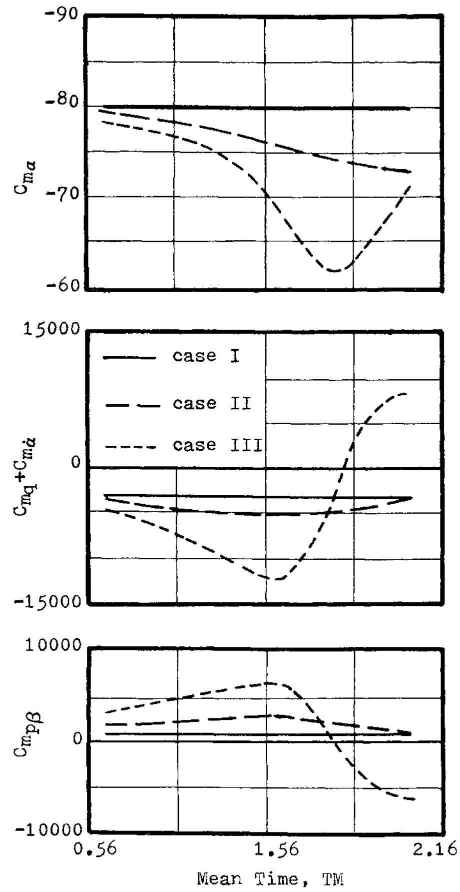


**Fig. 5 WOBBLE results on generated data.**

and the DIFLA routine, Table 2. For the symmetric data, Case I, both the WOBBLE and DIFLA results compare quite well. The nonsymmetric cases show where errors may occur when the improper model is used. While the DIFLA results predict stable motion, the WOBBLE results yield a nutation instability, $\lambda_f>0$. This results from the interpretation of nonsymmetric damping moments as being a Magnus type moment. This phenomena has been mentioned by Levy and Tobak[8] and Stone, et al.[10] However, to the authors' knowledge, this present study is the first to arrive at this conclusion by deterministic fitting techniques.

## Supersonic Cone Test Results

Yaw-time histories of two different cone configurations, a blunt 60° half-angle body and a sharp 10° half-angle body,

Table 2 WOBBLE and DIFLA results[a]

| | | Case I | Case II | Case III |
|---|---|---|---|---|
| $K_1$ | (D) | 22.60 | 30.30 | 35.88 |
| | (W) | 22.56 | 24.11 | 25.49 |
| $K_2$ | (D) | 18.94 | 13.42 | 9.01 |
| | (W) | 18.87 | 17.19 | 15.19 |
| $K_3$ | (D) | 22.60 | 21.86 | 18.90 |
| $K_4$ | (D) | 18.91 | 19.08 | 18.00 |
| $\lambda_1$ | (D) | −0.0995 | −0.0980 | −0.1134 |
| | (W) | −0.1003 | 0.0176 | 0.0984 |
| $\lambda_2$ | (D) | −0.3246 | −0.2902 | −0.2402 |
| | (W) | −0.3246 | −0.4987 | −0.7365 |
| $\omega_1$ | (D) | 12.17 | 12.01 | 11.91 |
| | (W) | 12.17 | 12.08 | 11.98 |
| $\omega_2$ | (D) | −11.17 | −10.95 | −10.67 |
| | (W) | −11.17 | −11.10 | −11.09 |

[a] D represents DIFLA results and W represents WOBBLE results.

were obtained using a free-flight wind tunnel technique.[2] This data was originally presented by Jaffe.[11] In his analysis, Jaffee was unable to obtain suitable results from his fitting routines and suggested that the symmetric force system of his mathematical model might be deficient. It was felt that these two sets of data might readily be handled by the techniques set forth in this study. For a more detailed description of the experimental techniques, see Ref. 11.

Since this data is similar to range data, the data was not given for even time increments. Thus, it was necessary to interpolate the data to obtain values at increments of 0.0005 sec. This was accomplished by means of second-order Lagrange polynomials using three tabular points. The small angle approximation were used to allow the conversion of the data to Euler angles, i.e., $\alpha=\theta$ and $\beta=-\psi$. It was felt that since the angular motion of both configurations was less than 20,° this approximation would not effect the eventual results. This approximation was made for all of the fitting routines and thus, the resulting comparisons should be valid.

The flight and physical parameters of the two data sets are presented in Table 3. Since the duration of the flights were quite small, less than 0.1 sec. the roll rates and Mach numbers may be assumed constant. The optical system used was found to possess accuracy to ±0.15° and Jaffe states that the accuracy of the data should be within ±0.2° for the 10° cone and ± to 0.5° for the 60° blunt cone.

Figure 6 presents a $\theta$-$\psi$ plot of the experimental data for the 10° cone. The solid line represents motion generated using the NIM results, the coefficients of which are presented in Table 4. Also presented in Table 4 are the linear results from the WOBBLE routine. The section lengths, NS, used in each fit are also presented. The WOBBLE results for 10° cone are for a section length of 2.3 cycles. This is the usual section length criteria for this fitting. Section lengths of 1.8 and 2.3 cycles failed to coverage for WOBBLE fits of 60° blunted cone. The section length used represents 1.3 cycles. The residuals produced by the fit were small as can be seen in the values for the probable error, PE, in Table 4.

A comparison for the 10° cone with the results of Walchner and Sawyer[1] for a nonspinning body is also presented in Table 4. Their tests were limited to angles of attacks less than the cone half-angle and thus symmetry is expected to be preserved.[10] However, only the average value of their coefficients is presented here. In presenting their results as functions of angle of attack, significant deviations from symmetric aerodynamics begin to appear at 8.° This only further encourages the dynamicst that nonsymmetric force models must be developed.

To further test the nonsymmetric nature of the 10° cone data, sectional fits using DIFLA were made. The results of the first three sections are presented in Table 5. The fits were made on identical section lengths for each routine. The probable error is of the order of the expected experimental error. Of interest is the fact that the $K_2$ and $K_4$ results differ from between 0.5° and 0.8°. Since this is two to four times the

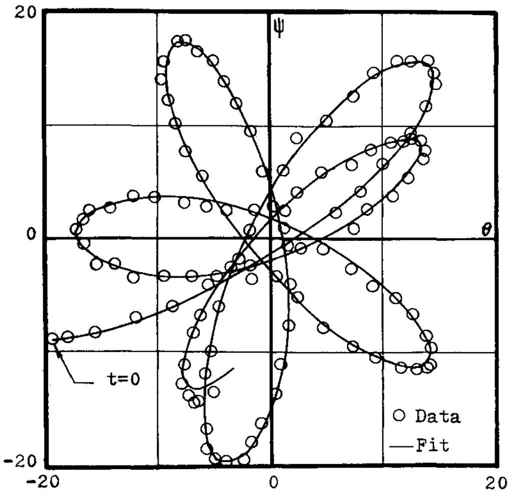


Fig. 6 Experimental data and NIM results for 10° cone.

Table 3 Physical parameters of supersonic cone models

| | 10° Cone | 60° Cone |
|---|---|---|
| mass (slugs) | $0.67\times10^{-3}$ | $0.19\times10^{-2}$ |
| $I$ (slug-ft$^2$) | $0.99\times10^{-6}$ | $0.49\times10^{-6}$ |
| $I_{xx}$(slug-ft$^2$) | $0.19\times10^{-6}$ | $0.73\times10^{-6}$ |
| Mach | 4.56 | 3.97 |
| $\rho$(slug-ft$^3$) | $0.16\times10^{-3}$ | $0.76\times10^{-4}$ |
| $V$(fps) | 4428.0 | 3855.0 |
| $d$ (ft) | 0.0825 | 0.1254 |
| $p$ (rad/sec) | 686.7 | 99.3 |
| $Re_d$ | $0.19\times10^{6}$ | $0.35\times10^{6}$ |

Table 4 Linear coefficient results for supersonic cone data[a]

| | | 10° Cone | 60° Cone |
|---|---|---|---|
| $C_{m_\alpha}$ | (N) | −0.190 | −0.046 |
| | (W) | −0.194 | −0.046 |
| | (Ref. 1) | −0.195 | |
| $C_{n_\beta}$ | (N) | 0.194 | 0.046 |
| | (Ref. 1) | 0.195 | |
| $C_{m_q}+C_{m_{\dot\alpha}}$ | (N) | −2.39 | −0.178 |
| | (W) | −2.23 | −0.506 |
| | (Ref. 1) | −2.30 | |
| $C_{n_r}-C_{n_{\dot\beta}}$ | (N) | −2.09 | −0.065 |
| | (Ref. 1) | −2.30 | |
| $C_{m_{p\beta}}$ | (N) | 0.447 | −0.327 |
| | (W) | 0.186 | 0.124 |
| $C_{n_{p\alpha}}$ | (N) | 0.051 | 0.533 |
| PE | (N) | 0.196 | 0.289 |
| | (W) | 0.180 | 0.122 |
| Section Length | (N) | 101.0 | 129.0 |
| | (W) | 71.0 | 61.0 |

[a](N) represents NIM results and (W) represents WOBBLE results.

**Table 5 Comparison of WOBBLE and DIFLA results-10° Cone[a]**

| Section Mean time | | 0.017 | 0.019 | 0.020 |
|---|---|---|---|---|
| $K_1$ | (D) | 10.35 | 10.38 | 10.36 |
| | (W) | 10.10 | 10.02 | 9.89 |
| $K_2$ | (D) | 8.93 | 8.99 | 9.02 |
| | (W) | 8.69 | 8.63 | 8.51 |
| $K_3$ | (D) | 9.92 | 9.99 | 10.20 |
| $K_4$ | (D) | 8.55 | 8.45 | 8.23 |
| $\lambda_1$ | (D) | −8.269 | −8.688 | −8.230 |
| | (W) | −7.261 | −6.915 | −6.600 |
| $\lambda_2$ | (D) | −5.989 | −5.913 | −5.098 |
| | (W) | −6.907 | −6.716 | −6.638 |
| $\omega_1$ | (D) | 436.6 | 435.8 | 434.6 |
| | (W) | 438.5 | 437.7 | 437.3 |
| $\omega_2$ | (D) | −304.9 | −304.9 | −304.0 |
| | (W) | −303.8 | −302.9 | −302.5 |
| PE | | 0.151 | 0.153 | 0.150 |
| | | 0.180 | 0.201 | 0.203 |

[a](D) represents DIFLA results and (W) represents WOBBLE results.

expected error of the data, it is felt that this difference is due to nonsymmetric moment components. Also note that the WOBBLE results predict a nutation damping rate greater than that predicted by DIFLA. This was also the case with the nonsymmetric generated data.

Since difficulties arose in the WOBBLE procedure for the 60° blunt cone, nonlinear coefficients were not obtained. However, a DIFLA fit over section lengths of 1.3 and 2.3 cycles showed $K_1$ and $K_3$ not to possess equal magnitudes. Again, this suggests the possibility of nonsymmetric aerodynamics.

## Conclusions

The fitting techniques presented herein provide viable numerical techniques for the extraction of aerodynamic coefficients from transient response data. The WOBBLE procedure is limited by its symmetric force model. The other methods, NIM and DIFLA, are applicable to nonsymmetric models.

With respect to computer time, both WOBBLE and DIFLA have similar run times. This is to be expected since they arise from similar algorithms. However, for systems in which many parameters (coefficients) are involved—such as those used in this study—the NIM technique requires much more time. Thus, a tradeoff situation arises when choosing which technique to apply.

An advantage of employing the DIFLA technique to nonsymmetric models is gained since DIFLA yields values of frequencies and damping rates directly. This technique could be used with various generic dynamic systems such as those encountered in flutter analysis. This method can be extended to nonlinear models by using asymptotic techniques. The use of NIM techniques for nonlinear models, which being quite straightforward, is extremely time consuming due to the large number of parameters.

The aerodynamic model employed herein represents a coalescence of linear aircraft theory and symmetric missile theory. While only linear values are obtained for a nonlinear phenomena, these values represent an "effective" coefficient and as such they can accurately describe vehicle motions. The use of these linear constitutive equations for force and moment reactions is justified by the nature of the dynamic flowfield about a spinning body of revolution.

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